Mathematical Modelling for Flood Risk Prediction and Urban Drainage Management in Nigeria: A Conceptual and Illustrative Framework

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Abstract
Flooding is one of the most destructive recurring environmental hazards facing Nigerian cities, yet much of the existing Nigerian literature on the subject remains descriptive rather than predictive. This paper sets out a conceptual and mathematical framework for predicting flood risk and supporting urban drainage management decisions in Nigeria, built around three complementary, well-established techniques: the Rational Method runoff equation, simple linear regression of rainfall against runoff, and the Gumbel extreme-value distribution for estimating rainfall return periods. After reviewing the conceptual, theoretical and empirical literature on flooding and mathematical modelling, the paper formally specifies each model and then works through a fully hypothetical numerical example, using assumed but literature-consistent parameter values, purely to demonstrate how the equations behave and what kind of planning insight they can yield once populated with real, site-specific data. The illustration shows, in principle, that peak runoff is highly sensitive to the proportion of impervious surface in a catchment, that rainfall intensity and runoff volume can be linked through a simple, testable statistical relationship, and that the estimated recurrence interval of extreme rainfall rises sharply, rather than proportionally, with event magnitude. The paper argues that this combination of classical hydrological and statistical tools offers Nigerian planning authorities an accessible, low-cost complement to more data- and computation-intensive approaches such as GIS-based terrain modelling and machine learning, provided the models are calibrated and validated against verified local rainfall and stream-gauge records before being used for actual design or policy decisions.

Keywords: *flood risk; mathematical modelling; urban drainage; Rational Method; Gumbel distribution; Nigeria*

1. Introduction

Flooding has shifted, in Nigeria, from an occasional seasonal nuisance to a recurring national emergency that affects almost every geo-political zone of the country (Umar & Gray, 2023). The devastating floods of 2022, triggered by a combination of unusually heavy rainfall and the release of excess water from the Lagdo Dam in Cameroon, displaced an estimated 1.4 million people and damaged infrastructure across more than ten states, illustrating how exposed Nigerian cities remain

to both climatic extremes and weaknesses in early-warning and drainage systems. Cities such as Lagos, Port Harcourt, Benin City, Lokoja, Makurdi, Ibadan and Bauchi continue to experience recurrent flooding driven by rapid, often unplanned urban expansion, the conversion of wetlands and floodplains into building sites, undersized or blocked drainage channels, and rainfall variability associated with climate change. Much of the institutional response to this problem in Nigeria has remained reactive, oriented towards emergency relief after a flood event rather than towards systematic, quantitative prediction. Mathematics offers a complementary route: by translating rainfall, catchment, and drainage characteristics into equations and statistical relationships, it becomes possible to estimate, ahead of time, how much runoff a given storm will generate, how that runoff relates statistically to rainfall intensity, and how often rainfall of a given severity is likely to recur. Internationally, such mathematical and computational approaches to flood prediction range from classical hydrological formulae to contemporary applications of Geographic Information Systems and machine learning (Komolafe, Awe, Olorunfemi & Oguntunde, 2020; Kafi, Ponrahono, Ash'aari & Barau, 2025). Within Nigeria specifically, however, the systematic application of even the classical mathematical and statistical tools to urban flood and drainage problems remains comparatively limited, with the literature still weighted towards descriptive accounts of flood impact (Umar & Gray, 2023). This paper addresses that gap at the conceptual and methodological level. It does not report results from a primary field survey or from verified agency datasets; instead, it formulates three complementary, well-established mathematical models relevant to flood prediction and drainage management, situates them in the relevant theoretical and empirical literature, and works through a fully hypothetical, clearly labelled numerical example to show how the models behave and what kind of planning insight a properly calibrated version of them could provide. The intention is to offer Nigerian researchers, students, and planning practitioners an accessible point of entry into mathematically grounded flood-risk assessment, while being explicit that the specific numbers used here are illustrative rather than empirical.

2. Literature Review

2.1 Conceptual Issues: Modelling, Flooding and Drainage

Mathematical modelling is the process of translating a real-world phenomenon into equations, functions or algorithms that can be analyzed, tested and used for prediction (Meyer, 2011). In environmental and hydrological work, such models are used to represent rainfall behaviour, flood intensity, and the performance of drainage infrastructure, with their reliability depending on the quality of input data and the appropriateness of the underlying assumptions. Flooding itself is the overflow of water onto land that is ordinarily dry, occurring when a river, stream or drainage channel is overwhelmed; in Nigeria it has become a major recurring socio-economic problem, with urban flooding especially severe in densely populated cities where drainage infrastructure is inadequate, poorly maintained, or obstructed by waste. Urban drainage management, in turn, covers the planning, construction and maintenance of systems that control stormwater and wastewater movement; as Nigerian cities urbanise, the replacement of permeable land with roads, pavements and buildings reduces natural infiltration and raises surface runoff, which is precisely the relationship that the models in Section 3 are built to capture.

2.2 Theoretical Framework

Three theoretical strands inform the modelling approach adopted in this paper. Hydrological theory explains the movement of water through rainfall, infiltration, runoff and discharge, and provides

the basis for the Rational Method, Q = CIA, in which peak runoff discharge Q is taken to be proportional to a runoff coefficient C, rainfall intensity I, and catchment area A; a complementary hydraulic relationship, Manning's equation, Q = (1/n)AR^(2/3)S^(1/2), links channel roughness, cross-sectional flow area, hydraulic radius and slope to the conveyance capacity of a drainage channel. Systems theory treats flooding as the outcome of interacting components, rainfall, drainage capacity, land use, topography and human activity, rather than of any single cause, which is the justification for examining land-use and rainfall scenarios jointly rather than in isolation (Section 4.4). Probability theory, finally, underpins the statistical estimation of how often extreme rainfall events recur; the Gumbel extreme-value distribution, with cumulative distribution function $F(x) = \exp[-\exp(-(x-u)/\alpha)]$, is widely used for this purpose, with the associated return period given by $T = 1/[1-F(x)]$.

2.3 Empirical Review

Nigerian empirical work has long identified inadequate drainage, poor waste disposal and uncontrolled urbanization as central drivers of urban flooding (Aderogba, 2012; Ogbonna & Uwajeh, 2014), while broader hydrological scholarship has established that mathematical and statistical models meaningfully improve flood prediction and disaster preparedness when properly combined with meteorological data (Chow, 2010; Todini, 2007; Smith, 2013). More recent Nigerian studies have extended this work computationally: Komolafe, Awe, Olorunfemi and Oguntunde (2020) combined multi-criteria analysis with a terrain-based hazard model to map flood vulnerability in the Ogun River Basin; Simeon, Edim and Eteng (2021) combined algorithmic logic with classical hydrological formulae to design a flood-magnitude prediction model; and Kafi, Ponrahono, Ash'aari and Barau (2025) applied machine-learning and explainable-AI techniques to flood risk in Bauchi, finding settlement formality, elevation, population and rainfall to be the most influential predictors. Reviewing the wider decade of Nigerian flood literature, Umar and Gray (2023) concluded that, while flood impact is well documented, the systematic application of mathematical and computational modelling to Nigerian flood data remains comparatively rare, particularly in the northern part of the country. The framework developed in this paper sits at the more accessible, classical end of this spectrum, complementing rather than competing with the GIS-based and machine-learning approaches above.

3. Model Formulation

3.1 The Rational Method

Peak runoff discharge is modelled as $Q = CIA / 360$, where Q is peak discharge in cubic metres per second, C is a dimensionless runoff coefficient between 0 and 1 that rises with the proportion of impervious surface in the catchment, I is rainfall intensity in millimeters per hour for a chosen design return period, and A is catchment area in hectares.

3.2 Rainfall-Runoff Regression

The statistical relationship between rainfall intensity (X) and runoff volume (Y) is modelled as the simple linear regression $Y = a + bX + e$, with a and b estimated by ordinary least squares and the coefficient of determination, R^2, used to assess how much of the variation in runoff is explained by rainfall intensity.

3.3 Gumbel Return-Period Model

The recurrence interval of an extreme rainfall magnitude x is estimated from the Gumbel cumulative distribution $F(x) = \exp[-\exp(-(x-u)/\alpha)]$, with location parameter u and scale parameter α obtained from the mean and standard deviation of an annual maximum rainfall series by the method of moments, and return period $T = 1/[1-F(x)]$.

4.1 Rational Method Illustration

A catchment of 250 hectares and a design rainfall intensity of 80 mm/hr. Table 1 shows how the runoff coefficient, representing different land-use conditions, changes the computed peak discharge.

C (assumed)	Hypothetical land-use condition	I (mm/hr)	A (ha)	Q (m^3/s)
0.3	Vegetated / low-density	80	250	16.67
0.5	Mixed residential	80	250	27.78
0.7	Dense residential/commercial	80	250	38.89
0.9	Highly impervious	80	250	50.00

Holding I and A fixed, the threefold rise in Q across the assumed C range illustrates, in principle, the sensitivity of peak discharge to land cover that the Rational Method predicts; an analogous real-world increase in impervious surface, without any change in rainfall, would be expected to raise actual peak discharge by a comparable order of magnitude.

4.2 Regression Illustration

Table 2 presents a set of ten rainfall-runoff pairs, constructed to fall within plausible literature ranges, purely to illustrate how the regression model in Section 3.2 would be fitted and read.

Obs.	X: rainfall intensity (mm/hr, assumed)	Y: runoff volume (m^3/s, assumed)
1	20	12
2	30	18
3	40	19
4	50	27
5	60	30
6	70	38
7	80	40
8	90	48
9	100	53
10	110	58

Ordinary least squares fitted to this hypothetical set gives $Y = 0.77 + 0.52X$, with $R^2 = 0.99$. The exercise is included only to show that, once real rainfall and runoff records are substituted, the same procedure would indicate the strength and slope of the rainfall-runoff relationship for an actual catchment; the very high R^2 obtained here reflects the smooth, noise-free way .

4.3 Gumbel Return-Period Illustration

Illustration, an annual maximum rainfall series with mean 85 mm and standard deviation 20 mm, the Gumbel parameters work out to $\alpha \approx 15.6$ and $u \approx 76.0$, giving the return periods in Table 3 for selected rainfall magnitudes.

x (mm, assumed)	F(x)	T (years)
100	0.807	5.2
120	0.942	17.3
140	0.984	61.3
160	0.995	219.0

The pattern illustrated here, a sharp rather than proportional rise in estimated return period with magnitude, is a general property of extreme-value distributions and would be expected to recur with any real rainfall series; it implies that, in practice, drainage infrastructure designed only against a short return-period storm would be expected to be exceeded comparatively often, which is consistent with the recurrent flooding documented in the empirical literature reviewed in Section 2.3.

4.4 Joint Scenario Illustration

Table 4 combines the C values from Section 4.1 with three design storms, again holding A at 250 ha, to illustrate how land-use and rainfall assumptions interact within the Rational Method.

Land-use (C, assumed)	I=60 mm/hr (Q)	I=90 mm/hr (Q)	I=120 mm/hr (Q)
Low-density (0.3)	12.50	18.75	25.00
Mixed urban (0.6)	25.00	37.50	50.00
Highly impervious (0.9)	37.50	56.25	75.00

In this matrix, the discharge produced by a comparatively common, lower-intensity storm under highly impervious land use (37.50 m^3/s) exceeds that produced by a much rarer, higher-intensity storm under low-density land use (25.00 m^3/s). The general points the illustration is built to make, that unmanaged urbanization can push the effective flood risk of a catchment up to a level otherwise associated with much rarer storms, is a plausible and literature-consistent implication of the Rational Method rather than a measured outcome for any specific city.

5. Discussion

First, the Rational Method implies that peak discharge is highly sensitive to land-use change, which is consistent with the qualitative findings of Aderogba (2012) and Ogbonna and Uwajeh (2014) that uncontrolled urbanisation is a major driver of Nigerian urban flooding, and suggests that this driver is, at least in principle, quantifiable and not merely descriptive. Second, the regression formulation in Section 3.2, , offers a comparatively simple and accessible way of testing and quantifying the rainfall-runoff relationship that more resource-intensive studies such as Simeon, Edim and Eteng (2021) and Kafi, Ponrahono, Ash'aari and Barau (2025) have approached through more elaborate algorithmic and machine-learning methods; the classical and the computational approaches are complementary rather than substitutes.

Third, the Gumbel illustration in Section 4.3 reflects a well-established statistical property of extreme rainfall, namely that the estimated return period escalates sharply with magnitude, which carries a clear practical implication for drainage design standards even before any specific Nigerian rainfall series is substituted in: the choice of design return period is a major lever in how often a piece of drainage infrastructure can be expected to be overwhelmed. Fourth, the joint scenario in Section 4.4 illustrates, in line with systems-theoretic reasoning (Section 2.2) and the spatial vulnerability findings of Komolafe, Awe, Olorunfemi and Oguntunde (2020), that land-use and rainfall assumptions interact rather than act independently, so that drainage and land-use planning are best treated as a single, joint decision problem rather than two separate ones.

6. Conclusion and Recommendations

This paper has set out a conceptual, classical mathematical framework, comprising the Rational Method, a rainfall-runoff regression model and a Gumbel return-period model, for predicting flood risk and informing urban drainage management in Nigeria, the framework is deliberately positioned as an accessible complement to, rather than a substitute for, more data- and computation-intensive approaches such as GIS-based terrain modelling and machine learning, and is intended primarily as a starting point for researchers, students and planning practitioners who wish to bring quantitative, mathematically grounded reasoning to bear on Nigerian flood and drainage problems.

7.0 Recommendation

On this basis, it is recommended that:

1. researchers and graduate students seeking to extend this work obtain verified rainfall and stream-gauge records for a specific Nigerian city and use them to calibrate and validate the three models presented here before drawing any empirical conclusions;
2. agencies such as the Nigerian Meteorological Agency expand and maintain the monitoring networks on which any such calibration would depend;
3. urban planning authorities treat land-use control and drainage-capacity planning as a joint decision problem, in view of the interaction illustrated in Section 4.4; and drainage design standards be periodically reviewed against updated return-period estimates rather than fixed historical assumptions, given how rapidly the consequences of under-design escalate with storm magnitude.
4. Future work should also explore the integration of the classical models presented here with the GIS-based and machine-learning approaches reviewed in Section 2.3, to combine the interpretability of the former with the spatial and predictive power of the latter.

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